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GUIDELINES FOR APPLICATION OF LEARNING/COST IMPROVEMENT CURVES

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16. Abstract <p>The materials that have been included in this document are completely within the state of the art and are oriented toward the types of programs that have become common at the Marshall Space Flight Center. The references have been selected to support this type of program, and the bibliography has been selected to provide greater depth of information. The content of Reference 2, "How to Use the Learning Curve" is still considered the best general reference for learning/improvement curves. If more detailed information is required, References 1 and 4 should be reviewed.</p> <p>The differences between the terms "learning curve" and "improvement curve" have been noted, as well as the differences between the Wright system and the Crawford system.</p> <p>Learning curve computational techniques have been reviewed along with a method to arrive at a composite learning curve for a system given detail curves either by the "functional techniques" classification or simply categorized by subsystem.</p> <p>Techniques are discussed for determination of the theoretical first unit (TFU) cost using several of the currently accepted methods. Sometimes TFU cost is referred to as simply "number one cost".</p> <p>A tabular presentation of the various learning curve slope values is given. The evaluation of more detailed information for learning curve slope values is currently underway. Hopefully, a more complete version of slope information can be published in the future.</p> <p>A discussion of the various trends in the application of learning/improvement curves and an outlook for the future are presented.</p>					
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GUIDELINES FOR APPLICATION OF LEARNING/COST IMPROVEMENT CURVES

SECTION I. INTRODUCTION

At Marshall Space Flight Center (MSFC) the term "learning curve" has taken on a broader meaning as will be outlined below. The term as it appears connotes a narrower meaning, i.e., "manufacturing assembly" or "repetitive clerical operations". If, however, it is intended to cover the more complex operations such as tooling, methods improvement, substitution of machines, improved product design, or process improvement, a more complex situation prevails and thus the term "improvement curve" has been used. Several other definitions have been applied, as itemized in Appendix A. At MSFC the terms "learning/improvement curve" have come into such popular general use in referring to the same general application that the terms will be continued as they are used now. The same general practice has been followed by many of the contractors, who also quite frequently refer to a process improvement within the broader context, but use the name "learning curve" to describe it.

In a logical sense learning curves may be thought of as progress toward a goal, with the ordinate being time or dollars per unit and the abscissa being units of production. When plotted on arithmetic coordinates, the result yields an exponential-type curve (decreasing) (Fig. 1). However, if plotted on log-log paper, the approximate result is a straight line (Fig. 2). Learning curves are generally specified by the slope of the curve in percent; i.e., an 80 percent curve means that each time the number of units produced is doubled, the doubled quantity will take only 80 percent as much time as the preceding group. In other words, there will be a 20 percent reduction in time units or direct labor hours [1]. For example, if units per lot are 1, 2, 4, 8, and 16, then hours/unit at 80 percent of the prior lot are 100, 80, 64, 51.4, and 41.2. Thus, for an 80 percent learning curve, the reduction for each doubled quantity of production would be 20 percent; for an 85 percent curve, 15 percent; and for a 90 percent curve, 10 percent [2].

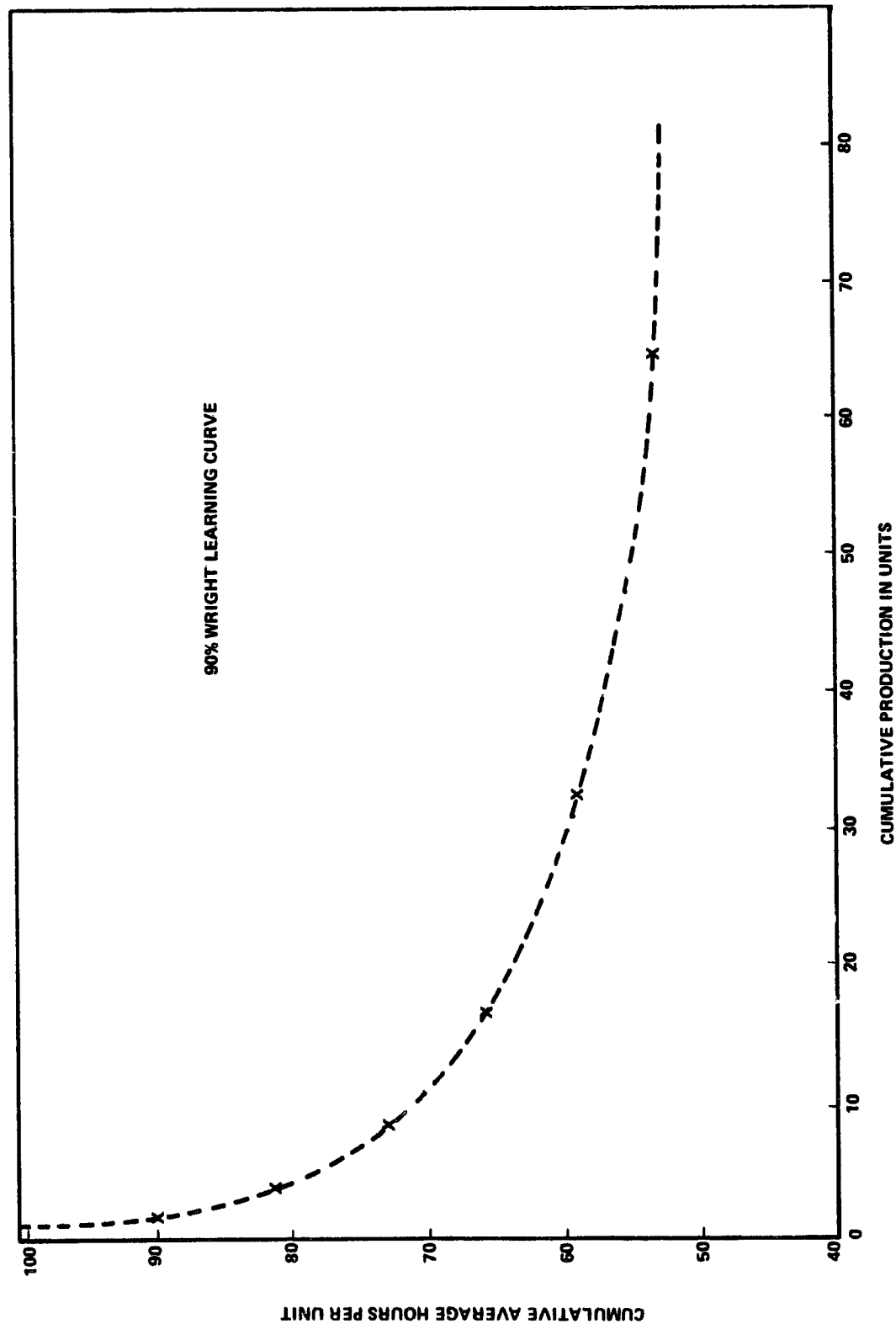


Figure 1. Plot of learning curve on Cartesian coordinates.

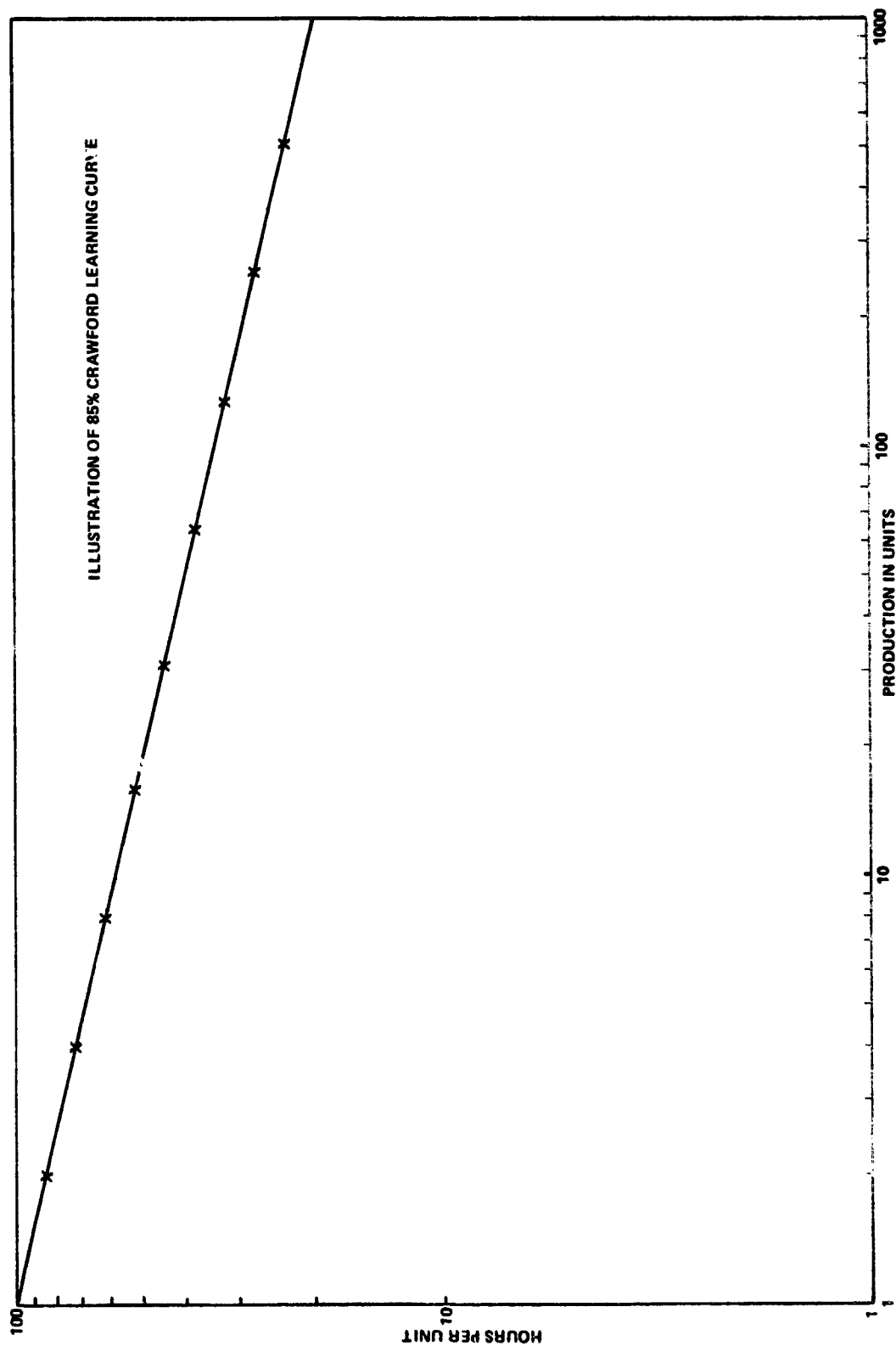


Figure 2. Plot of learning curve on log-log coordinates.

In general, the following types of factors contribute to the stated time reduction of a learning/cost improvement curve:

1. Operator learning
2. Improved methods, processes, tooling, and machines and improvements in design for manufacturability
3. Management learning
4. Debugging of engineering data
5. Rate of production
6. Design of the assembly or part, or modifications thereto
7. Specification or design of the process
8. Personnel factors such as fatigue, personal matters, or employee morale.

Materials costs also show improvement when plotted as a curve. Material improvement/discount curves average approximately 95 percent with a range from 90 percent to approximately 100 percent [3]. Since learning curves are usually steeper than material cost curves, those elements of materials cost that have a larger expenditure for labor should yield steeper slopes than the elements requiring less labor (as a part of their price).

Usually learning curve slope values refer to the straight line slope shown in Figure 2 and are usually given in percent, i.e., 80 percent or 90 percent. Also, the specification of a learning curve slope in percent should be considered as an estimate and not as a discrete value. Since the variables are continually changing (i.e., labor for assembly of a subsystem), we cannot expect any stated learning curve slope value to be more than an approximate value or range.

However, when working with data from a specific company based on its particular shop history for a process or piece of hardware with which the company has had experience, slope values for improvement curves can be accepted as authentic for the situation in question. It is the general application of learning curve slope information that creates a problem.

SECTION II. OBJECTIVES/STATEMENT OF PROBLEM

Questions have been asked frequently concerning the application of learning/improvement curves. A typical question would be as follows: "What should be the learning curve for an electronics subsystem?" As outlined below, one would need more information to give an intelligent answer to such a proposed hypothetical question. Such questions as those that follow would require answers:

1. How many units or subassemblies are involved?
2. What is the production rate?
3. Is the design of the subassembly within the state-of-the-art or a new concept?
4. What phase of the program are we talking about — DDTE or production phase?
5. What learning curve theory is being applied — Crawford, Wright, or other?

Often such information is readily available on projects that are under consideration at MSFC, but it is definitely needed before any meaningful answer can be given for the question as outlined above.

Data, as currently available, do not break down the learning/improvement curve application to a specific subsystem and/or component level. We are currently working toward this goal, but as of now it is not available.

Stated succinctly, the objective of this report is to state the goal as follows: "Provide a guidelines-type document for use by anyone seeking a ready reference on the application of learning/improvement curve information."

SECTION III. ASSUMPTIONS AND/OR CONSTRAINTS

As will be stated in more detail in a subsequent section, it will be assumed that MSFC will follow the Crawford system in learning curve/improvement curve applications, unless otherwise specified. As a part of this proposition, the learning curve as plotted on log-log paper will approximate a straight

line. The units for the ordinate axis will be the cost per unit in either man-hours or dollars per unit; the units for the abscissa will be number of units produced (Fig. 2). It will be assumed that a plot of coordinate points on log-log paper will approximate a straight line.

Since some contractors utilize the Wright system, this methodology is presented as an alternative method, and the proximate conversion may be effected as outlined in Appendix B. In the Wright approach the units for the ordinate axis are presented as the cumulative average cost per unit in either average man-hours or dollars per unit. In this approach it will be assumed that a plot of coordinate points on log-log paper will approximate a straight line.

SECTION IV. BACKGROUND: WRIGHT, CRAWFORD AND OTHER THEORY

A. Wright System

This system will be discussed first because it was the first system developed, dating back to the 1930's. In this approach it is assumed that a plot of coordinate points on log-log paper will approximate a straight line. The coordinate points will consist of the ordinate that is based on cumulative average cost in man-hours or dollars per unit and the abscissa which is the number of units produced or completed. Tables have been computed and are available for each percentage value of learning curve slope beginning with 60 percent up to a value of 99 percent. Each entry in the table gives values for the cumulative total, cumulative average, and unit values for each percentage point of the learning curve and for each unit of production up to 1000 units. In general the Wright system of plotting learning curve values will yield a smooth curve, since cumulative average values are used in plotting learning curve data. This is especially true when working with live or actual data as opposed to projected values.

B. Crawford System

The Crawford system for learning curve analysis is based on the assumption that the ordinate values are based on the unit values as opposed to a cumulative average of these values. The coordinate points for the Crawford system, or unit-cost system, are formulated such that the cost or value for each unit

only are plotted directly at the particular unit in question, i.e., the time or cost for the 10th unit, or the 30th unit, form the basis for the plot point. The plot point for the 10th unit on an 85 percent Crawford curve (Fig. 2) would be approximately 58 hours per unit. The coordinate values are based on the following: the ordinate values are the unit values in man-hours or cost in dollars per unit, and the abscissa values are based on the number of units produced or completed. Tables have been reproduced for each value of learning curve slope from 60 percent up to a value of 99 percent. Each entry in the table gives values for the cumulative total, cumulative average, and unit value for each percentage point of the curve and for each unit of production up to 1000 units.

Although most companies originally used the Wright system, in recent years many companies have adopted the Crawford system and at present approximately 93 percent of all firms utilize the Crawford or unit cost system [4].

C. Other Theory

In some cases a situation will arise that requires a modification to the basic theory (Wright or Crawford). Such is the case with a technique known as the Stanford system, which utilizes the "B-factor" to modify the basic curve. This B-factor is utilized to modify the curve to allow for prior experience or "knowhow" that a particular company might have. It is the number of equivalent units of any item theoretically produced prior to the production of the first actual unit. It is based on the assumption that a firm's prior experience will have the same effect on the improvement curve as if some equivalent number of a new product had already been produced. The B-factor, which is empirically based, usually is adjusted such that the B-factor runs from 1 to 6 for a particular type of hardware, i.e., structures, electronics, or power supply subsystem. The B-factor will increase from 1 to 6 as the firm's level of prior experience or background increases, with a B-factor of 0 indicating no prior experience. For example, one aerospace firm was allowed to utilize a B-factor to modify its basic learning curve to allow for prior experience in the production of a structures subsystem. In this system the cost of the first unit is depressed, depending on the value of the B-factor. This factor identifies at what level or unit of production the company will produce a unit value equivalent in cost to number one. If a company has a B-factor of 3, the cost of unit number four plotted on the basic learning curve will be the same as unit number one on a Stanford curve (Fig. 3). This allows a firm with prior experience to bid lower on the first unit or theoretical first unit (TFU) cost. After the TFU has been established, the remainder of the curve is plotted point by point to establish a smooth curve that will eventually merge with the regular learning curve as the plot is extended downward. The resulting plot on log-log paper is no longer a continuous straight line, but is an arched curve at the beginning and becomes straight as the production run is extended (5). Needless to say, the specification of B-factors will vary with each contractor and, if used, must be established through negotiation.



SECTION V. ANALYSIS OF THE DETERMINATION OF THE ESTIMATE OF THEORETICAL FIRST UNIT COST

If improvement curves are plotted from actual cost data, the slope and TFU may be determined from any two points on the curve. Estimates for a projected estimate of an improvement curve may be developed also, provided we have an estimate of the TFU and a projected curve slope. Since both the TFU and the rate of improvement (i.e., 80 percent) are estimates, this type of application must be used with great care. An error in judgement of ± 5 percent (curve slope) will affect the cumulative total cost of 1000 units by as much as 68 percent, depending on the slope of the particular curve.

The TFU cost is defined as the cost or resources required, whether in man-hours or dollars, of producing the first unit. It is called "theoretical" since rarely will the cost of producing the number one, or first unit, equate to the actual number of man-hours or dollars required to produce unit number one in a production sequence. In a phased sequence the development or test units are produced first and serve two purposes: to work out any design or development problems and to work out any production or manufacturing problems. Since in many cases these development units do not represent complete assemblies, they cannot be assumed to represent a production sequence. They do, however, provide experience and can be utilized in the process to develop cost data for the TFU in a production sequence.

In a process such as the above, the estimate for number one cost is used to determine the starting point for the improvement curve, and the curve is drawn from this point. This approach is complicated by working with an estimate of the TFU and also from using an assumed slope value for the improvement curve. Errors in one are compounded by the effects of the other.

In the more popular approach, the contractor begins by estimating a particular unit in the production sequence, say unit 30 or 100, draws an improvement curve through this point and then backs up the curve to find the cost of the TFU (Fig. 4). The decision as to what unit to use should be based on the type and complexity of the item and experience with the particular manufacturing facility. This is the preferred methodology.

Another technique for estimating the TFU is based on a company's previous experience. The typical vendor has had previous experience with production of a particular type of hardware. Usually, therefore, there is

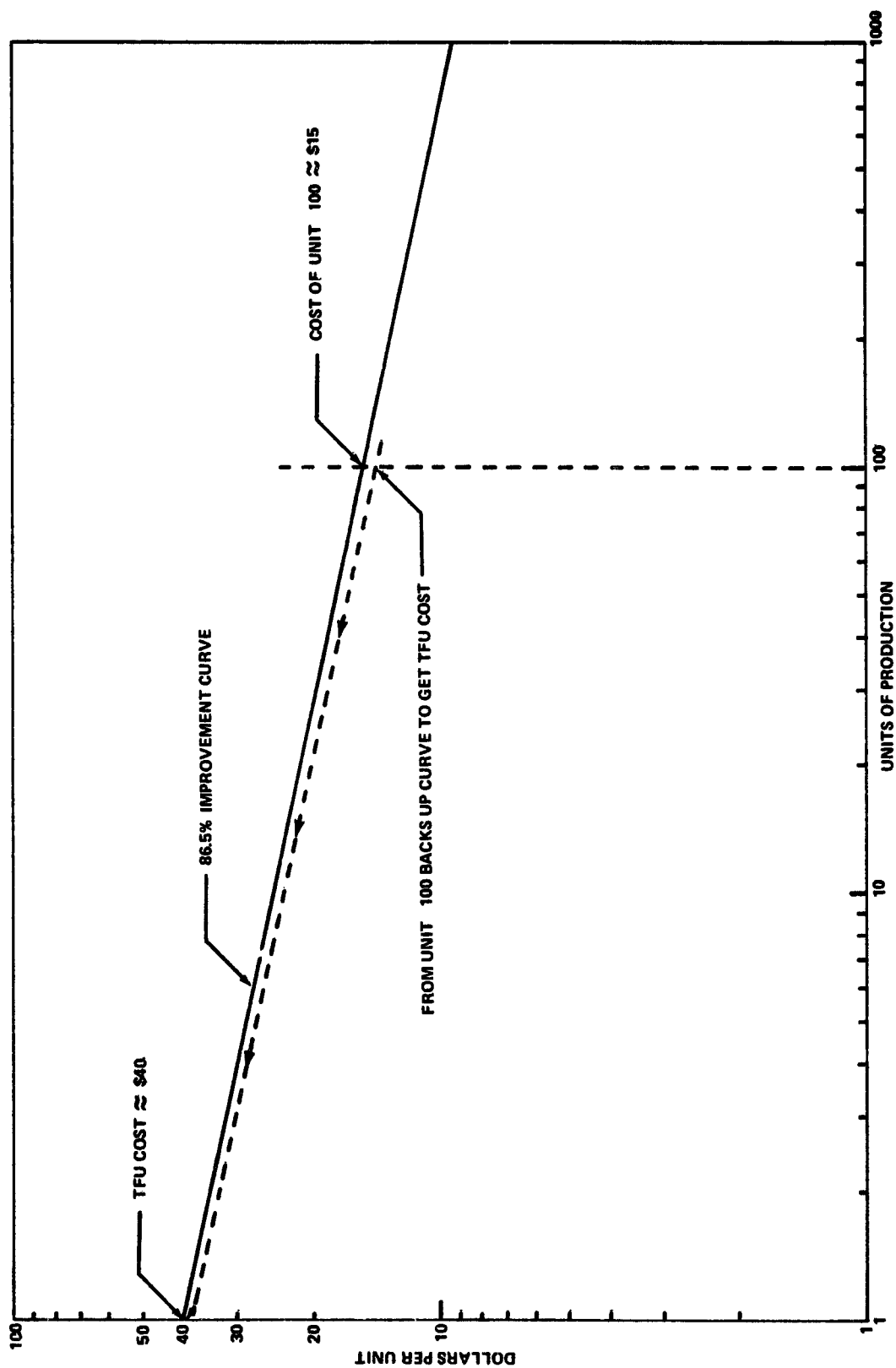


Figure 4. Determination of TFU using the 'backup' method.

ample historical data that can be used to compute the cost elements to produce a new item. This "estimate by comparison" usually involves the estimator and representatives of the various departments involved. Comparable hardware items, processes, etc., for which costs are known are selected as a guide. Elements of time and materials are deleted from, or added to, the cost estimate as required. Thus, the individual cost elements are priced for the subject configuration and a TFU cost is computed. This method requires a detailed step-by-step analysis of the differences in design and manufacturing process for the new item versus the previous experience.

SECTION VI. TABULATION OF VARIOUS COST IMPROVEMENT CURVE SLOPE VALUE RANGES

The slope values that are displayed herein are approximate values and could vary as much as ± 10 percent depending on the particular application and are dependent also upon the particular design in question (if we are considering a hardware situation). The improvement curve will tend to be higher in slope value (90 to 95 percent) for those cases that are automated to a greater extent, or for which tooling is what has been termed "hard" tooling, or where an automatic machine process is involved. Also, the job content, when examined in detail, will have an effect. If the job requirements contain principal elements that are repetitive in nature, the learning curve slope will be steeper, signifying a high percentage of learning. When a production rate is increased, the tendency will be for the slope of the improvement curve to flatten.

If a study is made for a situation involving an improvement curve analysis for a lower level package, some information must be available explaining the type of work or process being performed, e.g., machining, sheet metal work, or automatic electronics assembly. If the job contains partly handwork and partly machining, a rule of thumb must be applied to indicate a flatter improvement curve value as the percentage of machine time goes up. This relationship is outlined below.

There is an influence on the slope value for improvement curves based on the particular type of industry or industrial process that is involved, e.g., aerospace, ship building, chemical process, construction, etc. It is realized that curve values for these industries may not be directly related, but the slope values are listed nevertheless for comparison purposes.

Improvement curve slope overall values for industry in general are as follows:

1. Aerospace — 85 percent
2. Shipbuilding — 80 to 85 percent
3. Complex machine tools for new models — 75 to 85 percent
4. Repetitive electronics manufacturing — 90 to 95 percent
5. Repetitive machining or punch-press operations — 90 to 95 percent
6. Repetitive clerical operations — 75 to 85 percent
7. Repetitive welding operations — 90 percent
8. Construction operations — 70 to 90 percent
9. Raw materials — 93 to 96 percent
10. Purchased parts — 85 to 88 percent.

Factors that will in general contribute to an improvement curve time reduction are as follows:

1. Improved methods, or method innovations
2. Process improvement or time reduction
3. Improvements in design for increased manufacturability
4. Debugging of engineering data
5. Rate of production
6. Introduction of a machine to replace a hand operation.

Machine-paced operations for production of machined parts such as screws, engine blocks, or gears and carefully engineered manual processes used in the assembly of small electronic parts on a printed circuit show little or no learning after the initial setup.

There is a rule of thumb relative to the proportion of assembly direct labor hours to machining direct labor hours for the slope of the improvement curve as follows:

1. 75 percent assembly/25 percent machining — 80 percent
2. 50 percent assembly/50 percent machining — 85 percent
3. 25 percent assembly/75 percent machining — 90 percent
- An individual operator doing the recurring work on a new product using standard manufacturing methods — 85 percent
- Total labor effort (setup cost, repair of the inspection rejects, nonrecurring activities, etc.) — 80 percent.

Crawford versus Wright system assumptions are as follows:

1. Constant unit cost (Crawford) assumption always results in a higher total cost estimate.

2. If an estimator assumes the cumulative unit cost method (Wright) and the unit cost (Crawford) is true, the resulting estimate will be understated (too low).

3. In recent times the application of unit cost theory (Crawford) has been used in 93 percent of the cases.

The following examples of subsystem or component part improvement curves have not been substantiated but are given as reference material only:

1. Fabrication — 85 to 95 percent
2. Subassemblies — 75 to 85 percent
3. Final assembly — 75 to 80 percent
4. Structures subsystem — 84 to 88 percent
5. Electronics subassembly — 85 percent
6. New state-of-the-art product — 75 to 80 percent

7. Standard electronic parts — 95 percent
8. Fabricated parts, standard catalog items — 90 percent
9. Labor, learning only — 85 percent
10. Insulation application — 85 percent
11. Refurbishment — 85 percent
12. Solid motor propellant — 95 percent
13. Igniter (solid motor) — 90 percent
14. Electrical fabrication — 90 percent.

SECTION VII. DESCRIPTION OF VARIOUS COST IMPROVEMENT CURVE ANALYTICAL TECHNIQUES

A. Summary of Analysis Methods

The solution of cost improvement curve problems falls roughly into categories of three types of solution: (1) graphical method, (2) solution by use of the tables, and (3) solution by the use of formulas. The solution whether for the Wright or Crawford systems is made generally by the same techniques that are illustrated in the following sample solutions. Any solution requires basically the same information and may be given with the following types of data*: either the slope of the improvement curve and the first unit cost, or the cost of a specific unit and the slope of the curve. Even when solving a problem by the analytical method or by use of the tables, it is usually advisable to check the solution using the graphical method.

* Also, if two or more points are known, the log-log plot of an improvement curve may be plotted on log-log paper.

B. List of Improvement Curve Formulas

1. $Y = A \cdot X^{-b}$

or

$$\log Y = \log A - b \log X \text{ (computing form) ,}$$

where

Y = dollars or man-hours per unit.

A = first unit cost or time.

X = number of units, or unit number.

b = geometric slope of acute angle that log linear learning curve makes with horizontal.

2. $b = 3.31895 \log_{10} m = \text{geometric slope} ,$

where

m = slope given in percent (i.e., 80 percent) for a log-linear learning curve.

Note: The values for geometric slope b are shown in Table 1.

3. $Y = CA \cdot (1 + b) = \text{unit value in dollars or man-hours per unit} .$

$CA = Y/(1 + b) = \text{cumulative average in dollars or man-hours per unit} .$

4. $A = Y/X^{-b} = \text{first unit cost or time} .$

$$\log A = \log Y + b \log X .$$

TABLE 1. IMPROVEMENT/LEARNING CURVE SLOPE TABLE

Slope, m, %	Angle B, in degrees	-b (tan B)	1 + b	1/1 + b
100	0	0	1.0	1.0
99	0.9	0.01571	0.98429	1.01596
98	1.8	0.03143	0.96857	1.03234
97	2.7	0.04716	0.95284	1.04949
96	3.6	0.06291	0.93709	1.06713
95	4.5	0.07870	0.92130	1.08542
94	5.4	0.09453	0.90547	1.10440
93	6.3	0.11040	0.88960	1.12410
92	7.2	0.12633	0.87367	1.14460
91	8.1	0.14232	0.85768	1.16594
90	9.0	0.15838	0.84162	1.18818
89	9.9	0.17453	0.82547	1.21143
88	10.8	0.19076	0.80924	1.23573
87	11.7	0.20679	0.79321	1.26070
86	12.6	0.22353	0.77647	1.28788
85	13.5	0.24008	0.75992	1.31593
84	14.4	0.25676	0.74324	1.34546
83	15.3	0.27357	0.72643	1.37660
82	16.2	0.29053	0.70947	1.40950
81	17.1	0.30764	0.69236	1.44434
80	18.0	0.32492	0.67508	1.48131
79	18.9	0.34238	0.65762	1.52064
78	19.8	0.36002	0.63998	1.56255
77	20.7	0.37787	0.62213	1.60738
76	21.6	0.39593	0.60407	1.65544
75	22.5	0.41421	0.58579	1.70710
74	23.4	0.43274	0.56726	1.76286
73	24.3	0.45152	0.54848	1.83220
72	25.2	0.47056	0.52944	1.88879
71	26.1	0.48989	0.51011	1.96036
70	27.0	0.50953	0.49047	2.03886
69	27.9	0.52947	0.47053	2.12526
68	28.8	0.54975	0.45025	2.22099
67	29.7	0.57039	0.42961	2.32769
66	30.6	0.59140	0.40860	2.44738
65	31.5	0.61280	0.38720	2.58264
64	32.4	0.63462	0.36538	2.73688
63	33.3	0.65688	0.34312	2.91443
62	34.2	0.67960	0.32040	3.12110
61	35.1	0.70281	0.29719	3.36485
60	36.0	0.72654	0.27346	3.65684

$$5. Y = F \cdot A \text{ (tabular solution) ,}$$

where

F = factor value from table at a certain number of units and slope.

A = cost in man-hours or dollars for the first unit.

Y = dollars or man-hours per unit.

C. Sample Solutions by Various Techniques

Solutions by the graphical method are as follows:

1. Assume we are given a problem with the following information: Wright system, unit number one cost is \$ 30 (A), and the slope of the improvement curve is 80 percent. We are required to find the cumulative average of the value of unit number 80. The answer from the attached graphical solution (Fig. 5) is approximately \$7 per unit.

2. Another type of problem is given when the slope of the improvement curve is unknown, but the following information is known for a Crawford curve: the unit number 1 value is known to be \$40; the unit number 4 value is known to be \$25.60; the unit number 8 value is known to be \$20.48; and the unit number 20 value is known to be \$15.25. These four points are plotted on log-log paper and the slope of the curve is measured (Fig. 6). The answer from the attached graphical solution (Fig. 6) is approximately an 80 percent curve slope.

Tabular solutions are as follows (Table 2):

1. Given: Wright improvement curve problem; slope = 83 percent and first cost = \$250.

To Find: Cost of unit 100, or Y ,

$$Y = F \cdot A$$

Factor from table at 83 percent = 0.289979 .

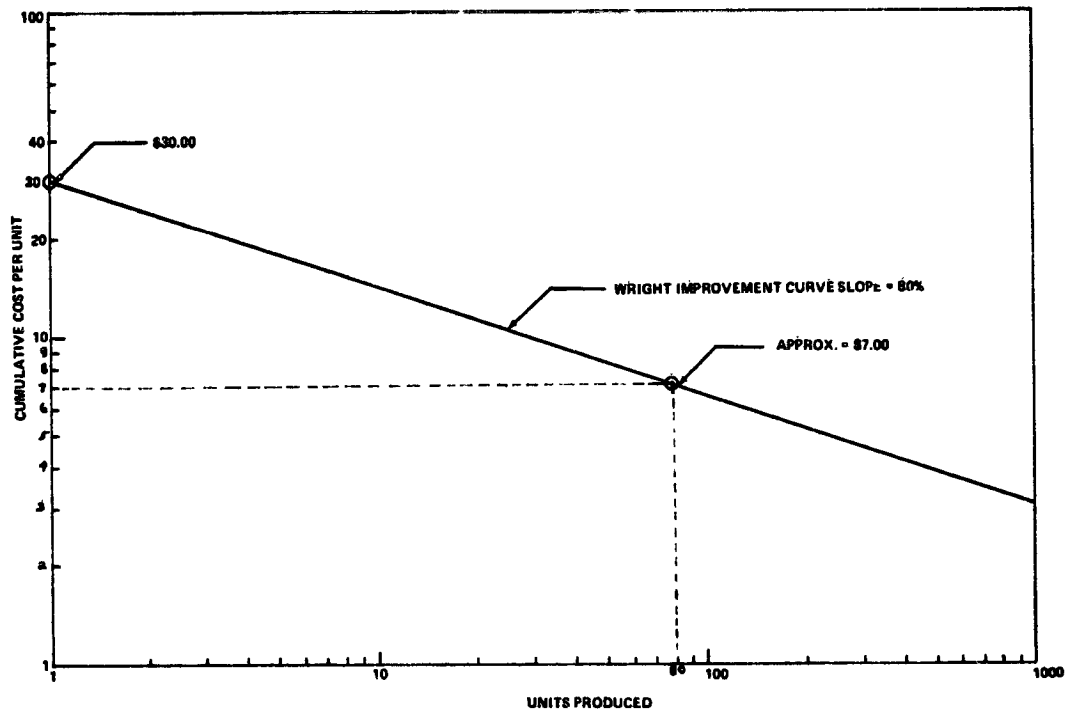


Figure 5. Solution of improvement curve problem using graphical technique.

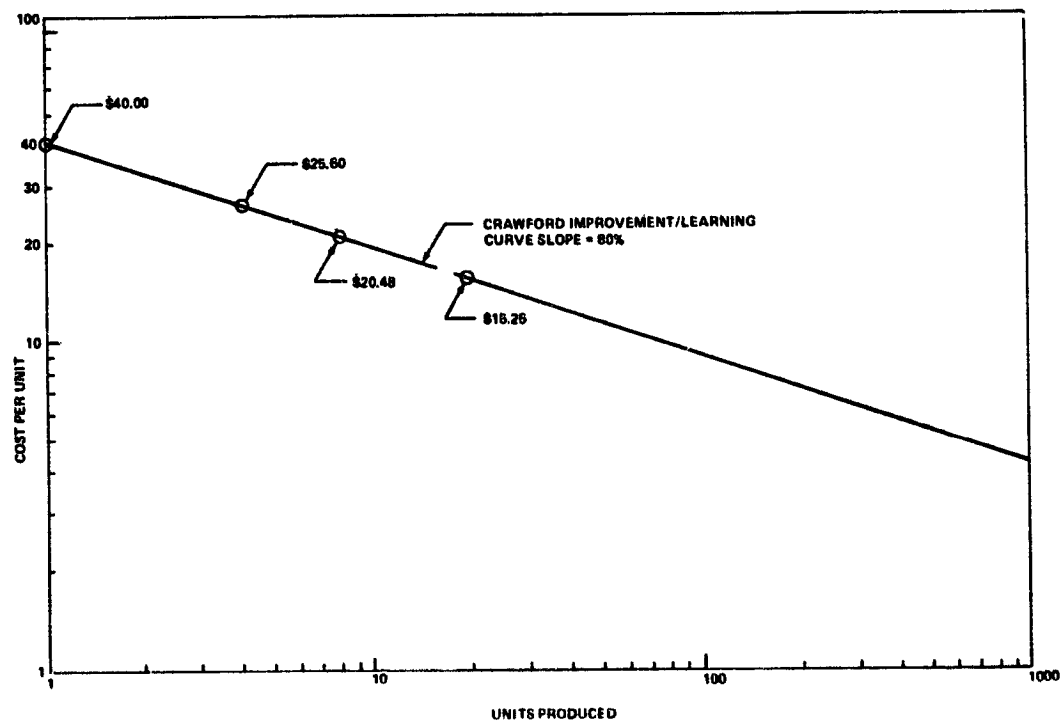


Figure 6. Solution of improvement curve problem when given 4 points.

TABLE 2. SUMMARIES OF WRIGHT AND CRAWFORD DATA

A. Slope-Quantity Factors for Crawford System

Item	Slope	Quantity															
		2	3	4	5	7	10	25	50	100	250	500	1000	2000			
Log-Linear Unit Curve (C)	95	0.950	0.922	0.903	0.888	0.866	0.843	0.788	0.749	0.711	0.665	0.631	0.600	0.670			
	90	0.900	0.846	0.810	0.783	0.744	0.705	0.613	0.552	0.497	0.432	0.389	0.350	0.315			
	85	0.850	0.772	0.723	0.686	0.634	0.583	0.470	0.400	0.340	0.274	0.233	0.198	0.188			
	80	0.800	0.702	0.640	0.596	0.535	0.477	0.355	0.284	0.227	0.169	0.135	0.108	0.0866			
	75	0.750	0.634	0.563	0.513	0.446	0.385	0.263	0.197	0.148	0.101	0.0758	0.0569	0.0427			
	70	0.700	0.569	0.490	0.437	0.367	0.306	0.191	0.134	0.0935	0.0584	0.0408	0.0286	0.0200			
Cumulative Average Curve (A)	65	0.650	0.505	0.423	0.368	0.298	0.239	0.135	0.0879	0.0571	0.0323	0.0210	0.0137	0.0088			
	95	0.975	0.957	0.944	0.932	0.915	0.895	0.844	0.804	0.766	0.716	0.681	0.647	0.615			
	90	0.950	0.915	0.899	0.868	0.835	0.799	0.708	0.643	0.581	0.508	0.457	0.412	0.371			
	85	0.925	0.874	0.836	0.806	0.762	0.712	0.692	0.510	0.438	0.355	0.303	0.258	0.213			
	80	0.900	0.834	0.786	0.747	0.690	0.632	0.492	0.402	0.327	0.246	0.198	0.159	0.127			
	75	0.875	0.794	0.737	0.692	0.626	0.559	0.408	0.315	0.242	0.168	0.127	0.0961	0.0723			
	70	0.850	0.756	0.690	0.639	0.566	0.493	0.336	0.246	0.178	0.114	0.0818	0.0574	0.0405			
	65	0.825	0.718	0.644	0.589	0.510	0.434	0.276	0.191	0.130	0.0771	0.0514	0.0340	0.0224			

B. Slope-Quantity Factors for Wright System

Item	Slope	Quantity															
		2	3	4	5	7	10	25	50	100	250	500	1000	2000			
Log-Linear Unit Curve (C)	95	0.900	0.866	0.844	0.829	0.806	0.784	0.731	0.694	0.659	0.615	0.585	0.555	0.528			
	90	0.800	0.739	0.701	0.675	0.638	0.602	0.521	0.469	0.421	0.366	0.330	0.297	0.267			
	85	0.700	0.619	0.571	0.538	0.494	0.452	0.362	0.307	0.260	0.210	0.178	0.152	0.129			
	80	0.600	0.506	0.454	0.418	0.371	0.329	0.242	0.193	0.154	0.115	0.0917	0.0734	0.0597			
	75	0.500	0.402	0.348	0.314	0.269	0.230	0.155	0.116	0.0867	0.0592	0.0444	0.0333	0.0250			
	70	0.400	0.305	0.255	0.224	0.185	0.152	0.0936	0.0652	0.0455	0.0284	0.0198	0.0139	0.00972			
Cumulative Average Curve (A)	65	0.300	0.216	0.174	0.149	0.118	0.0935	0.0519	0.0335	0.0217	0.0123	0.00796	0.00517	0.00336			
	95	0.950	0.922	0.903	0.883	0.866	0.843	0.788	0.749	0.711	0.665	0.631	0.600	0.570			
	90	0.900	0.846	0.810	0.783	0.744	0.705	0.613	0.552	0.497	0.432	0.389	0.350	0.315			
	85	0.850	0.773	0.723	0.686	0.634	0.583	0.470	0.400	0.340	0.274	0.233	0.198	0.168			
	80	0.800	0.702	0.640	0.596	0.535	0.477	0.355	0.284	0.227	0.169	0.135	0.108	0.0866			
	75	0.750	0.634	0.563	0.513	0.446	0.385	0.263	0.197	0.148	0.101	0.0758	0.0569	0.0427			
	70	0.700	0.568	0.490	0.437	0.367	0.306	0.191	0.134	0.0935	0.0584	0.0408	0.0286	0.0200			
	65	0.650	0.505	0.423	0.368	0.298	0.239	0.135	0.0879	0.0571	0.0323	0.0210	0.0137	0.0088			

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Therefore,

$$Y_{100} = (0.289979) (250)$$

$$Y_{100} = 72.4475 \text{ or } \$72.45 \text{ per unit.}$$

2. Given: Crawford improvement curve problem; slope = 93 percent,
 $X = 70$, and $Y = \$250$.

To Find: A or cost of first unit,

$$Y = F \cdot A, \quad A = Y/F$$

$$\text{Factor from table at 93 percent} = 0.640948 .$$

Therefore,

$$A = 250/0.640948$$

$$A = \$390 .$$

Solutions by use of formulas are as follows:

1. Given: Wright improvement curve problem; slope = 85 percent and
first unit cost = \$50 (A) .

To Find: Cost of unit number 60, or Y.

Solution:

$$Y = A \cdot X^{-b} \text{ (general form)}$$

$$\log_{10} Y = \log_{10} A - b \log_{10} X \text{ (computing form)}$$

$$\log Y = \log (50) - .24008 \log (60)$$

$$\log Y = 1.69897 - (0.24008) (1.77815)$$

$$\log Y = 1.69897 - 0.4268982$$

$$\log Y = 1.27207$$

$$Y_{60} = 18.709 \text{ or } \$18.71 \text{ per unit.}$$

2. Given: Crawford improvement curve problem; slope = 90 percent, X = 200, and Y = \$65 per unit.

To Find: A, or cost of first unit.

Solution:

$$A = Y/X^{-b}$$

$$\log_{10} A = \log_{10} Y + b \cdot \log_{10} X$$

$$\log A = 1.812913 + (0.15838) (2.30103)$$

$$\log A = 1.812913 + 0.364437$$

$$\log A = 2.177350$$

$$A_1 = 150.4352 \text{ or } \$150.44.$$

The methodology for computation of a composite improvement/learning curve is simply a procedure to weight each subtask or subsystem curve in proportion to a dollar value or time of the individual subelement. The approach for this method is as follows:

$$M_c = \sum [V_{ss}/T] M_{ss}$$

and

$$M_p = [V_{ss}/T] M_{ss} ,$$

where

M_c = composite learning curve slope

M_p = proportionate part of learning curve slope

V_{ss} = value of subsystem or subtask in time or dollars

T = total time or dollars for system

M_{ss} = slope of curve for subtask or subsystem expressed by a whole number, i.e., 85 percent.

Sample problems for this methodology are as follows:

1. Given: Subtask time = 4.483 minutes = V_{ss} ; total time for job = 54.55 minutes = T ; and slope for subtask = 79 percent = M_{ss} .

$$M_p = [V_{ss}/T] M_{ss}$$

$$M_p = [4.483/54.55] 79$$

$$M_p = 6.4923 \text{ percent.}$$

2. Given: Thrust vector control system manufacturing: subtask — final assembly, cost = \$100,000 = V_{ss_1} and improvement curve = 80 percent = M_{ss_1} ; subtask — electronics, cost = \$300,000 = V_{ss_2} and improvement curve = 93 percent = M_{ss_2} ; subtask — structures, cost = \$200,000 = V_{ss_3} and improvement curve = 85 percent = M_{ss_3} ; and total manufacturing cost = \$600,000 = T .

$$M_c = \sum [V_{ss}/T] M_{ss}$$

or

$$M_c = [(100,000/600,000) 80 + (300,000/600,000) 93 + (200,000/600,000) 85]$$

$$M_c = 1/6 (80) + 1/2 (93) + 1/3 (85)$$

$$M_c = 13.33 + 46.5 + 28.33$$

$$M_c = 88.16 \text{ or } 88.2 \text{ percent.}$$

A diagram depicting how the subtasks of the problem fit together is shown in Figure 7.

SECTION VIII. DISCUSSION

A. Present Practice

The current application of learning/improvement curves has been divided for the most part between the Crawford and Wright systems, although the Crawford is reportedly the more popular system (93 percent of cases). Tables have been developed for both systems and are readily available.

Learning curves have been utilized primarily to describe overall systems or programs. These have been reported in the literature primarily to describe overall fabrication processes, e.g., aerospace, 85 percent; electronics, 93 percent; or repetitive machining, 90 to 95 percent. Regardless of the approach or published value of the curve slope, considerable care should be taken since in reality all of the curve slopes are approximations at best. They should always be treated in this manner since a learning/improvement curve must be projected for each different design and process of manufacturing. When a projection is made for such a unique design, there is always a certain amount of uncertainty in the specification of an improvement curve. Based on the currently available information on improvement curves, estimates should be treated as a forecast and not as exact criteria.

B. Future Trends

Recently there has been a trend in procurements away from the practice of listing an improvement curve for an entire project. On the Shuttle program, contracts have been let separately for such items as the external tank, rocket motor, recovery system, etc. Learning/improvement curves have been listed

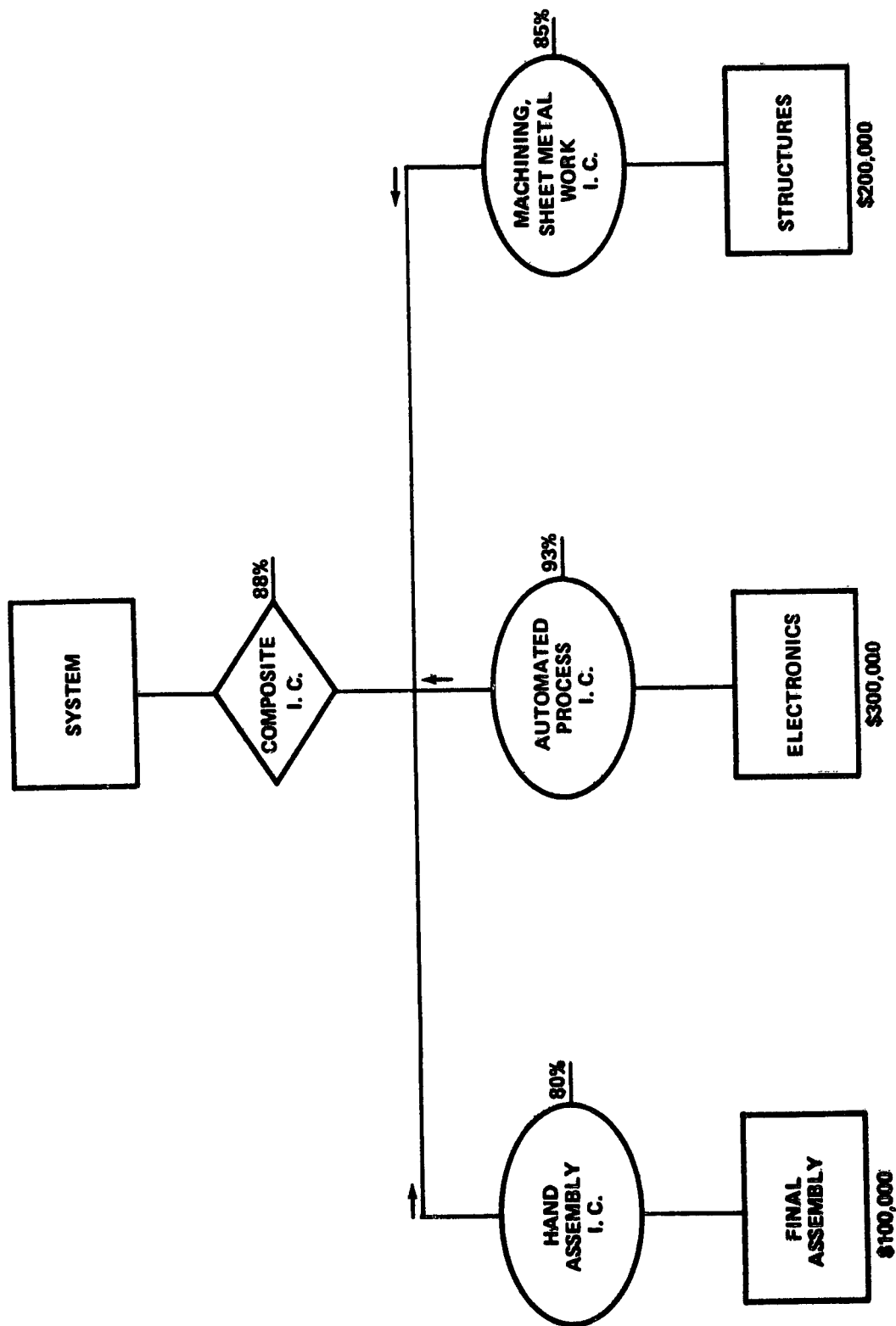


Figure 7. Diagram of composite improvement curve.

for each of these subsystems and/or components. The problem is, of course, that published information is not available at this level of detail in the literature. The government at present does not have any source available to verify the accuracy of the information that is furnished in the various cost proposals. Regardless, the trend is in the direction of subsystems and/or components for learning/improvement curve specification.

Also, in the determination of the TFU, or theoretical first unit cost, the trend is away from the technique of trying to determine the cost elements of the first unit by estimating the cost elements "before the fact". Usually, by the time unit number 20, unit number 30, or unit number 100 is produced, it is easier to estimate a cost for the unit. A theoretical first unit cost is then determined by laying out the cost of, for example, unit number 100 at a learning curve slope of some given value on log-log graph paper and backing up the learning curve to obtain the value of the TFU (Section V).

By the same token present trends indicate the use of a "composite" curve made up of all learning/improvement curves at a lower level. The technique for combining the lower level curves is illustrated in Section V. The composite curve should represent the combination of all lower level learning/improvement curves.

APPENDIX A

GLOSSARY OF TERMS

1. Cumulative Average Rule (CAR) — This term refers to an approach or ground rule to be used in plotting and/or analysis of learning or progress functions (curves). If this rule is used, cost or man-hour values for each production unit are used to compute a cumulative average time or dollar value starting with the second unit. These cumulative average values are then used to plot learning or progress curves. A curve slope based on the cumulative average rule may be observed in RFP's, or cost proposals, and specified in percent (e.g., 80 percent). Such quotations based on the CAR cannot be interchanged with those based on the unit rule, or some other mode of analysis. A synonym for "CAR" is the "Wright System".

2. Design Complexity (DC) — This form of complexity has to do with features or parameters of an engineering design which contribute to its complexity. Examples of such features which tend to increase the measure of design complexity are such aspects as total number of parts, number of fasteners, or number of subassemblies. Others might be the number of different steps or processes required to fabricate, assemble, and inspect.

3. Design Configuration Type (DCT) — A design configuration type is a term used to designate a category or generic class of system configurations for which both the technical and cost parameters could be expected to be typical. When estimating costs of large systems, example DCT's would be solid propellant boosters, nuclear powered submarines, army tanks, or jet airliners. Such examples represent rather distinct examples of large system types, each of which is made up of a unique set of subsystems and hardware components.

4. Factor — This term can be considered a synonym of parameter as far as this research is concerned.

5. Improvement Function (Curve) (IF) — This term is often used to describe the performance aspects of a system or design over time which tend to improve. Since both learning and other changes in a system performance may be included, it cannot be considered as a synonym for "learning curve" but may be interchanged with the term "Progress Function" which is listed in the following.

6. Job Specification (JS) — This term refers to the qualifications, performance/experience requirements, skill, and/or education that a prospective candidate must have in order to qualify for a particular job. Usually there is a corresponding job/position description that defines the duties, functions, and responsibilities which a candidate would be expected to perform.

7. Job/Task Design (JD) — This term refers to the total activity of planning and specifying all of the necessary steps, tools, equipment, environmental requirements, and/or any other performance criteria required for a qualified operator to perform.

8. Job/Task Environment (JE) — All of the atmospheric or comfort requirements which are necessary for a worker to successfully perform his job. Included would be lighting, heating, cooling, ventilation, safety and health needs, and, in some cases, acoustical or structural dynamics attenuation.

9. Learning Curve (LC) — A learning curve is a graphical plot on either Cartesian or double logarithmic paper that represents the rate of learning progress by humans, usually in performance of some task or group of tasks. In the engineering discipline this plot is usually made with time as the ordinate parameter and number of units complete or simply number of units as the abscissa. In general, these curves will approximate an exponential shaped function, if the progress is normal. This function should be separated from progress and improvement functions by the fact that only human learning progress is to be included in a learning curve — not tooling, design, or other gains in performance which may be a part of progress or improvement functions.

10. Log-Linear — This term is often used to describe learning curves which are plotted on double-logarithmic paper. In general, such curves will appear as straight lines. This greatly simplifies computation of the slope and will, of course, make these curves easier to plot.

11. Material Discount Curves (MDC) — This term refers to curves which are used to project the decrease in the cost of material and many purchased items, as the quantity of the item purchased is increased. Sometimes tables are used to reflect this information, and also double logarithmic paper is used since this function will frequently have a shape similar to a learning curve and will appear linear on double-log paper.

12. Model — A model is an approximation of reality which is frequently used to forecast or predict performance approximations of real world situations. Models may be physical or analytical within this context. Analytical models are

sometimes referred to as math models, or as algorithms which consist of a necessary and sufficient set of terms, values, and formuli needed to compute or predict an output value based on known input or set of input values and recognized constraints or limitations.

13. MTM — This is an acronym used to refer to a type of time study values that are determined by reference to standard tables, as opposed to making actual time studies of a job or task. The specific words are Methods Time Measurement or MTM.

14. Operator Performance Rate (OPR) — This term refers to a performance rating given a worker by an observer which is relative to a standard time or standard output rate for a particular job or task. If the person is performing at a speed which is, for example, 20 percent above normal, his rating would be 120 percent. Conversely, if the individual is performing at a speed which is 20 percent below normal, he would be given a rating of 80 percent.

15. Parameter — For purposes of this study, the terms factor, design feature, or parameter may be used interchangeably. A parameter is a term which is used to measure or gauge some feature or physical characteristic of a system or design. This measure is usually defined in some unit that is officially accepted, such as weight in grams or volume in cubic feet, etc.

16. Producibility (P) — This system specialty parameter refers to inherent capability or characteristics that enable a system to be manufactured, inspected, and/or checked out.

17. Product Assurance (PA) — Product Assurance is a system specialty factor which combines several of the subfactors such as reliability assurance, quality assurance, and safety assurance. PA includes all activities that directly or indirectly support or increase the likelihood that a product or system will perform its intended function in accordance with established criteria, standards, specifications, or other requirements.

18. Progress Functions (PF) — This term refers to the class of functions which, although related to learning curves, cannot be interchanged since a progress function should include all improvements, maturations, learning or other advances in technology or management which would tend to reduce resource requirements over time.

19. Subtask — A subtask refers to a separate part of a job or task; in other words, one of several procedural steps required to complete an activity.

20. Unit Rule (UR) — This term refers to an approach or ground rule to be used in plotting and/or analysis of learning or progress functions (curves). If this rule is used, cost or man-hour values for each production unit are plotted directly instead of using average or cumulative average values. A learning curve slope quoted in percent based on use of the unit rule cannot be interchanged with one which is based on the cumulative average rule (see definition above). A synonym is the "Crawford System".

21. System Specialty Parameters (SSP) — These are expressions of system performance variables or characteristics concerned with the overall technical effectiveness of an integrated system. System specialty parameters are used in system modeling, system trade studies, technical performance measurements, and assessments. Typical examples of specialty parameters are reliability, availability, maintainability, safety, survivability, etc.

22. Weighting Coefficients — These values are usually expressed in fractional parts and are used to transfer the desired emphasis to alternative performance ratings or estimates of value. The sum of such weights must always equal 1; if whole numbers are preferred, the sum must equal 10. If there is no particular emphasis desired by the decision maker, then each alternative will receive an implied weight of one.

23. Wright System — This term is a synonym for CAR or cumulative average rule (see item 1).

APPENDIX B

GRAPHICAL COMPARISON OF WRIGHT VERSUS CRAWFORD CURVES

When comparing the Wright system to the Crawford system, it may be observed that for the same curve slope and unit number the values for cumulative average cost for the Wright system and unit cost for the Crawford system are the same. In effect if we plot a curve for these values, it will be a common line. This is illustrated in Figure B-1. It may be observed that for the Wright system the unit value curve falls below the cumulative average line. The plot for the unit cost value is the common line in Figure B-1. For the Crawford system, the cumulative average curve falls above the common line.

The only problem with the above approach lies in the fact that the units for the Crawford system ordinate values are cost per unit, whereas the basic units for the Wright system are cumulative average cost per unit as indicated in the tables. Since the two systems are built on a different data base, the conversion from one to the other cannot be made on a direct basis. However, one can observe from reference to a table that the Wright system cumulative average value is exactly equal to the Crawford system unit value (see Table 2 in Section VII).

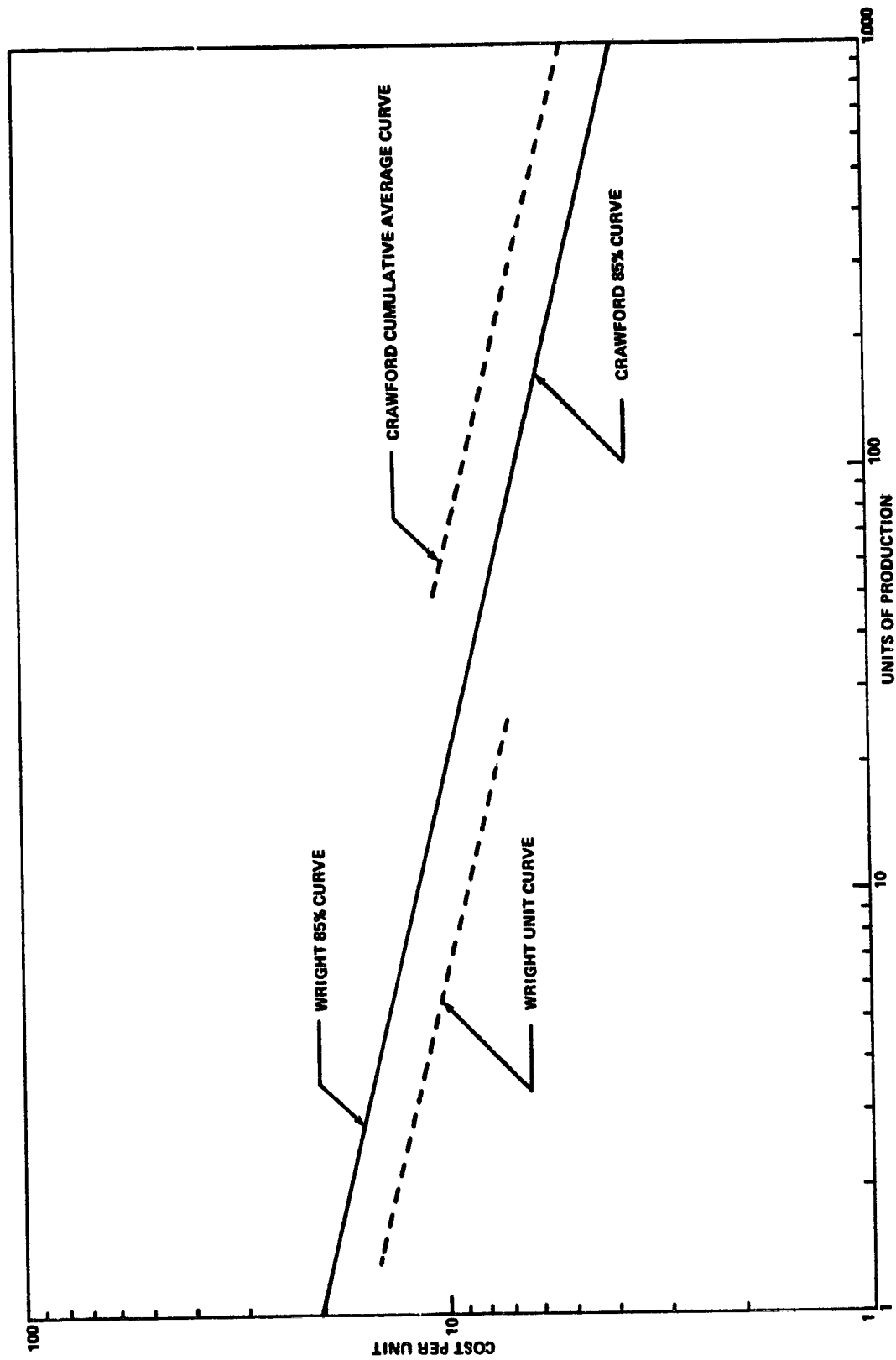


Figure B-1. Graphical comparison of Wright versus Crawford curves.

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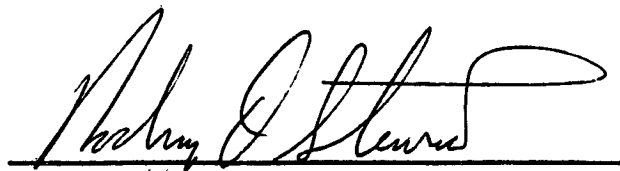
APPROVAL


GUIDELINES FOR APPLICATION OF LEARNING/COST IMPROVEMENT CURVES

By Dr. Leon M. Delionback

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.



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